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A PROBABILISTIC TRUTH-CONDITIONAL SEMANTICS FOR INDICATIVE CONDITIONALS¹

SUMMARY: In my article, I present a new version of a probabilistic truth prescribing semantics for natural language indicative conditionals. The proposed truth conditions can be paraphrased as follows: an indicative conditional is true if the corresponding conditional probability is high and the antecedent is positively probabilistically relevant for the consequent or the probability of the antecedent of the conditional equals 0. In the paper, the truth conditions are defended and some of the logical properties of the proposed semantics are described.

KEYWORDS: indicative conditionals, conditional probability, connection intuition.

1. Introduction

In my article, I will present a new version of a probabilistic truth prescribing semantics for natural language indicative conditionals. In this introductory section, I will present the basic notions and ideas which will be helpful in the rest of the article. In the second section, I will present the natural predeces-

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sor of my theory—Adams’ probabilistic theory of indicative conditionals. The third section will be devoted to the new version of the theory itself. In the fourth section, I will discuss the problem of compound conditionals (e.g., “If Martha is in the kitchen, we will have dinner soon, and if Marv is in the garage, the car will be fixed tonight”). The fifth and final section will discuss some of the implications of my theory.

1.1. Indicative Conditionals

Defining indicative conditionals is not an easy task. I will do it by defining counterfactual conditionals which constitute the complement of indicative conditionals in the set of all conditionals. Some examples of counterfactual conditionals are:

- C1 If he had not tampered with the machine, it would not have broken down. [(165)a.]
 C2 He would make more progress if he were using a computer. [(164)b.]

Typical counterfactual² conditionals share two features:

- CM1 The counterfactual conditionals use “would” as the auxiliary of its main verb.
 CM2 A speaker who uses a counterfactual conditional implies that the antecedent is false.

Conditionals that do not share one of these characteristics will be called indicative conditionals. From now on whenever I will write “conditional” I mean an indicative conditional.

Here are some examples:

- (1) If it rains a lot, the ground will become waterlogged. [(77)a.]³
- (2) People burn (instead of tanning) if they have a white, freckled skin. [(85)c.]
- (3) If you press this button, the fire alarm goes off. [(548)c.]
- (4) If the witness is prepared to testify, we have a strong case against Harry Field. [(548)c.]
- (5) If he does not say anything, he will not betray us. [(553)a.]
- (6) If his daughter is beautiful, my daughter is a Venus! [(653)b.]

² For the summary of the discussion about the demarcation of conditionals, including examples of less typical counterfactuals, see the work of Bennett (2003), which is also the source of CM1 and CM2.

³ Most of the examples that I use come from (Declerck, Reed, 2001). In the brackets, I will place the number of examples from the book. In case I modified the example I will mark it by adding * to the number in the bracket.

- (7) People ignore the warning if you do not point out the consequences. [(919)a.]
 (8) If you won all the fights, I am Cassius Clay. [(587)b.]

By theories of conditionals I mean theories that define the semantic value (truth or acceptability) of simple conditionals on the basis of some properties of its antecedent and consequent.⁴ Probabilistic theories of conditionals define the semantic value of conditionals on the basis of the conditional probability of the consequent given the antecedent.

1.2. Connection Intuition

Almost everything about conditionals is controversial. At the same time, it seems that there is at least one widely shared intuition: every positively valued conditional involves the existence of a connection between antecedent and consequent. Before further elaboration, the intuition has to be restricted, for not all of the conditionals used by competent speakers involve such a link. For example, (8) does not involve any kind of connection. Still, conditionals such as (8), so-called Dutchman conditionals, are sufficiently rare and specific to not disqualify the intuition. I will call the main body of conditionals, which involve the connection, *canonic conditionals*. The Dutchman conditionals will be discussed in the third section.

Further elaboration of the notion of connection is a tricky task. If we define it too narrowly, some positively valued canonic conditionals will be left outside. If we define it too broadly, the connection thesis becomes a trivial one. More than that just a few examples (1)–(7) show that in some cases the connection is hard to specify. In cases like (1), (2), or (3) the categorization is quite straightforward: the connection is clearly causal.

On the other hand, in the case of (6), it is not quite clear what the nature of the link is. Still, there are ways to argue for the existence of such a connection even in such cases. For example, we can claim that every situation which would justify the utterance of such conditionals has to involve some link, such as the following:

Two women, Jane and Alice, talk about the countenance of the daughter of their common friend, Susan. Jane claims that Susan is beautiful. On the basis of that statement, Alice reasons about Jane's aesthetic taste and her criterion of beauty. On the basis of both, she infers that Jane would also categorize her (Alice's) daughter as beautiful and claims (6).

⁴ Historically the first theory of conditionals in that sense was the theory of the material implication. It identifies truth conditions of natural language conditionals with truth conditions of material implication. The theory suffers from many counterexamples and, despite many defense attempts, seems to be disqualified (for examples of the defense of the theory, see Ajdukiewicz, 1956; Grice, 1989; see Edgington, 1995 for criticism).

So even in the case of (6), there seems to be some, in this case inferential, connection between the antecedent and the consequent.

Perhaps despite the diversity of conditionals, some additional characterization of the connection common to all true canonical conditionals could be given. For example, the inferential theories of conditionals (e.g., Douven, Elqayam, Krzyżanowska, 2022; Krzyżanowska, Wenmackers, Douven, 2014) leverage the fact that the connections between antecedents and consequents of true conditionals can be unpacked in a form of valid arguments. For example, in the case of (6) the argument can resemble the inference made by Alice. This feature of the connection inherent to true conditionals was developed into the following truth conditions:

Definition 1. A speaker S 's utterance “If p , q ” is true iff (i) q is a consequence—be it deductive, abductive, inductive, or mixed-of p in conjunction with S 's background knowledge, (ii) q is not a consequence—whether deductive, abductive, inductive, or mixed—of S 's background knowledge alone but not of p on its own, and (iii) p is deductively consistent with S 's background knowledge or q is a consequence (in the broad sense) of p alone (Krzyżanowska, Wenmackers, Douven, 2014, p. 5).

The theory is surely promising, on the other hand, Definition 1 contains many concepts meaning of which is still controversial. For example, it is unclear which logical system determines when a deductive argument in question is valid. The same goes for inductive and abductive arguments. These gaps, acknowledged in (Douven, Elqayam, Krzyżanowska, 2022), will undoubtedly be filled in the future, and these developments will likely lead to plausible, fully-fledged theories of conditionals. In this paper, I will develop a different way of conceptualizing the connection. I will use the fact that the link between antecedents and consequents of many conditionals is positive and probabilistic. By probabilistic I mean merely that we can capture that connection in probabilistic terms. By positive I mean that the occurrence of what is described by the antecedent makes more likely the occurrence of what is described by the consequent. It is easy to see that the links involved in our examples (stated in proper contexts) meet these two requirements.

I will show that a version of probabilistic semantics is able to capture the connection by exploiting these two properties. I present two versions of probabilistic semantics: in the next section, the classical theory by Adams, and in the third section a new proposal.

2. The Traditional Version of the Probabilistic Theory and Its Problems

By “traditional probabilistic theory of conditionals”⁵ I mean the proposal developed by Ernest Adams in his (1975). It has received a lot of attention

⁵ I will use TPC for brevity.

and approval, some of the most influential defenders of the theory are Dorothy Edgington (e.g., 1995), David Over (e.g., Over, Cruz, 2018) or Jonathan Bennett (e.g., 2003).

TPC does not aim at defining truth conditions of conditionals. It denies that conditionals have truth values at all. Instead, it defines their acceptability conditions. Acceptability (Ac) is understood here as aptness to be rationally accepted. So when is a conditional acceptable? According to the thesis called “Adams’ thesis”:⁶

$$AT \quad Ac(A \rightarrow B) = Pr(B/A), \text{ provided } Pr(A) \neq 0$$

As we see, AT equals the acceptability (Ac) of a conditional with the conditional probability of its consequent given its antecedent ($Pr(B/A)$). Conditional probability can be defined in different ways depending on, among other things, which probability theory we use. Adams used the standard Kolmogorov calculus and the standard definition of conditional probability (sometimes called the Ratio Formula):

$$RF \quad Pr(B/A) = \frac{Pr(B \wedge A)}{Pr(A)}$$

With all that in place, wherever we know the distribution of probability for the A and B we can compute how assertable is a conditional that involves these two sentences as antecedent and consequent ($A \rightarrow B$ or $B \rightarrow A$). For example, let us say that I want to know how high is the assertability of:

(9) If you jump from the fourth floor balcony, you will break your legs.

and we know the corresponding probability distribution (let us say $Pr(9a) = 0.01$ ⁷ and $Pr(9a \wedge 9c) = 0.0095$) we can compute it in following way:

$$\begin{aligned} Ac(9a \rightarrow 9c) &= Pr(9c/9a) \\ Pr(9c/9a) &= \frac{Pr(9a \wedge 9c)}{Pr(9a)} \\ \frac{Pr(9a \wedge 9c)}{Pr(9a)} &= \frac{0.0095}{0.01} = 0.95 \end{aligned}$$

So:

⁶ By \rightarrow I mean the functor which connects an antecedent and a consequent in a natural language conditional.

⁷ I will use xc and xa to refer to the consequent and antecedent of the example number (x). In the case of compound conditionals of the form, e.g., $(yaa \rightarrow yac) \rightarrow yc$ I will use, e.g., yaa to refer to the antecedent of the embedded conditional. It is easy to see that for example in the case of y , ya denotes $yaa \rightarrow yac$, I will use in such cases both labels interchangeably.

$$\text{Ac}(9a \rightarrow 9c) = 0.95$$

This means that our conditional is highly assertable. A qualitative version of AT defines the categorical acceptability of conditionals:

(QAT) An indicative conditional “If A , B ” is assertable for/acceptable to a person if and only if the person’s conditional degree of belief, $P(B|A)$, is high.⁸

As we see, QAT provides a threshold of conditional probability above which the conditional is acceptable. If we accept 0.95 as high, (9) is judged by QAT to be acceptable. It is a good prediction.

2.1. Problems

TPC gives us many similarly correct results. On the other hand it has some problems, for example:

Problem 1. The denial of truth-aptness of conditionals causes many problems. For example, the reactions of participants of the experiments assessing the truth value of conditionals suggest that conditionals have truth values. When asked to assess such values (e.g., Douven, Elqayam, Singmann, van Wijnbergen-Huitink, 2020; Krzyżanowska, Collins, Hahn, 2017) they are not confused and react to conditionals as to any other truth-apt sentence. This is, unexpected if conditionals are not propositions, consider for example asking somebody about the truth-value of a clearly no truth-apt sentence, for example, a question. Such a question would be at the very least confusing. Similarly, proposals that deny the truth-aptness of conditionals have trouble with explaining embeddings of conditionals; it will be discussed in the fourth section. Other deficiencies of such an approach are discussed, for example, in (Douven, 2015; Hájek, 2012).

Problem 2. Another problem are cases of incorrect predictions. In a case where two sentences are probabilistically independent (the probability of one of the sentences does not depend on the truth of the other) the conditional probability of the consequent given the antecedent is equal to the unconditional probability of the consequent. Therefore in such a case, if we took a conditional with very probable consequent and independent antecedent, the effect of the computation would be high acceptability. That seems unintuitive. To see this let us consider an example:

(10) If I eat an apple today, I will not inherit 1000000\$ today.

⁸ This formulation is inspired by one from (Douven, Verbrugge, 2012, p. 483).

As far as I know, sadly, there is no prospect of me inheriting any dollars, so the probability of $10c$ is high, let us say 0.99. The probability of $10a$ is quite high too, let us say 0.5 (I eat an apple every other day). We can also safely assume that $10a$ does not influence $10c$ probabilistically. so $\Pr(10a \wedge 10c) = \Pr(\sim 10a \wedge 10c) = 0.495$. Now we can compute $\Pr(10c/10a)$ by RF and $\text{Ac}(10a \rightarrow 10c)$ by means of AT:

$$\Pr(10c/10a) = \frac{\Pr(10a \wedge 10c)}{\Pr(10a)} = \frac{0.495}{0.5} = 0.99$$

So, (10) is highly acceptable according to TCP, which does not correspond to our intuitions. We typically do not accept such conditionals as true and if stated they would be seen as misleading.

3. A New Theory

In this section, I will propose a version of a theory that will not suffer from the problems of TPC.

Before I do that I want to note that the theory is, in a way, an idealization. The adequacy of the parameter (0.75) of the truth conditions (TC) which are the core of the theory has not been empirically tested. Therefore it is not clear how empirically adequate my proposal is. At the same time I believe that on the basis of some empirical tests, a more adequate version of my theory could be formulated. Such tests would involve subjects to make linguistic decisions involving conditionals in probabilistically transparent situations.

Another issue we should mention here is that the threshold may be sensitive to the pragmatic circumstances of the utterance. For example, a conditional whose consequent describes a dangerous event may require a lower threshold to be true. This effect can also be tested but it seems that it should be described by a pragmatic rather than a semantic theory of conditionals.

In Section 3.2 I will define the general truth conditions for conditionals and then apply them to some examples.⁹

I will assume that every sentence expresses a unique proposition,¹⁰ which can be represented by a set of possible worlds. The probability of a proposition is the probability that it is true. By the probability of a sentence, I will understand the probability of the proposition corresponding to that sentence. It seems that ascribing probability directly to sentences would not influence the rest of my work.

⁹ Their application to the cases of compound conditionals will be discussed in the fourth section.

¹⁰ By assuming that I am simplifying by ignoring context dependence, ambiguity, etc.

3.1. Truth-Conditions

The core of the new theory is the definition of truth conditions for simple conditionals, namely:

TC The conditional $A \rightarrow B$ is true iff:

- a) $\Pr(B/A) > 0.75$ and
 $\Pr(B/A) > \Pr(B)$
 or
- b) $\Pr(A) = 0$
 and false otherwise.

Both clauses of TC aim to capture different types of true conditionals.

a) captures canonical conditionals. The first conjunct aims to capture the sufficient correlation between what is described by a consequent and an antecedent of the conditional. As I already noted in face of the lack of empirical studies it is not clear how empirically adequate is the value of 0.75. The second conjunct of a) prevents TC from classifying, as true, conditionals with a non-relevantly high conditional probability (10-like). Therefore it is easy to see that (10) is not true according to TC. On the other hand, all conditionals (1) to (7) are classified as true.

The second clause is directed at Dutchman conditionals or more technically the “Ad absurdum” inferentials,¹¹ their classical examples are:

(11) If this is not a genuine piece of 17th century Japanese pottery, I’m a Dutchman. [(584)]

(12) If you are the new Messiah, I am Napoleon. [(587)a.]

or (8). These are not the cases of the canonic conditionals, there is no connection involved. The way we use such sentences suggests that when we use them we state that the antecedent is impossible. Consider for example:

Q1 Do you think that it is possible that it is not a genuine 17th century Japanese vase?

A1 It is impossible.

Now, if somebody uses (11) instead of A1 his answer would be equally adequate. It would preserve the original meaning. If so, we can easily incorporate

¹¹ Label is taken from (Declerck, Reed, 2001).

truth conditions for Dutchman conditionals by means of b).¹² Interestingly, it seems that the inferential semantics will have an analogous feature. If we assume that impossible sentences are inconsistent with the background beliefs of the speaker then, because of the explosion principle valid in classical deductive logic, we have a valid argument from an impossible antecedent to any sentence.

The consequence of adopting these truth conditions for Dutchman conditionals is that most of them are literally false. We usually use them with antecedents that are not, strictly speaking, impossible. For example, in the case of (11), it is difficult to see why “This is not a genuine piece of 17th century Japanese pottery” should actually be impossible. Still, it seems that we use expressions like A1 in similar cases where they are also literally false, so the consequence seems to be unproblematic.

How should we interpret the probability used in the definition? The definition is compatible with both subjective and objective interpretations of probability. Which of the two interpretations will be more natural depends on how realistic one is about conditionals. If somebody leans toward the suppositional view, claiming that conditionals express subjective degrees of beliefs of the speakers, then unsurprisingly the subjective interpretation of probability is natural. If somebody prefers the more objective interpretation according to which by uttering conditionals we want to claim, for example, something about regularities present in the external reality then the objective interpretation of the probability seems to be more appropriate.

There is another important issue to note here. We sometimes use conditionals with past sentences as arguments.¹³ In such cases, it is natural that these sentences are actually true or false and that the probability of a false one is 0 while the probability of a true one is 1. If we combine that with TC then we will obtain a very problematic result: all conditionals with false antecedents satisfy subclause b) and are therefore true. To avoid this unwanted consequence we have to introduce a small adjustment to the theory. In the case of such conditionals, we have to use hypothetical probability instead of the actual one. We can obtain it by suspending belief in the truth of a given sentence and imagining how probable it is. In a similar way can obtain conditional probability which is needed to judge the truth value of a given conditional.

How does the theory work then? If we want to know whether the conditional is true, we compare the corresponding probability distribution with TC, first, we

¹² Another way to explain the uses of Dutchman conditionals would be to treat them as rhetorical conditionals, one which does not aim to be literally true. I have two reasons not to do it. Firstly, it seems to me that these conditionals are used in a very systematic way. That made them easy to incorporate into TC. Secondly, they naturally fill the gap in RF. It does not give us any result for conditionals with impossible antecedents.

¹³ A famous example is: “If Oswald did not shoot Kennedy, someone else did” (Adams, 1970).

check if clause b) is satisfied,¹⁴ and if it is not, we check if clause a) is. If one of the clauses is satisfied by a probability distribution, the corresponding conditional is true, and if none of them is satisfied it is false.

3.2. Examples

Let us see some applications of the proposed theory.

Example (3). Let us assume that in the case of (3), the probability that the button is pushed is 0.5.¹⁵ If (3) is true it must be stated about the button which is responsible for turning on the alarm. The alarm has to be reliable so the probability that it goes off without pushing the button is low $\Pr(\sim 3a \wedge 3c) = (0.01)$, it is also very probable that if the button is pushed the alarm will start $\Pr(3a \wedge 3c) = (0.49)$. The probability of $3a$ is not 0 so following the procedure we have to compute the conditional probability of $3c$ given $3a$:

$$\Pr(3c/3a) = \frac{\Pr(3a \wedge 3c)}{\Pr(3a)} = \frac{0.49}{0.5} = 0.98$$

So the first part of the first clause of TC is satisfied. What about the second one? The probability of the consequent is 0.5, which is less than 0.98, so the second part of the first clause is also fulfilled, and so (3) is true.

Example (12). In all true (12)-like conditionals, the probability of the antecedent is 0. For example, in the case of $12a$, a true messiah is impossible. Thus the second clause is satisfied, therefore the conditional is true.

These results seem to be correct. Furthermore, my theory will judge (7) but not (10) as true. In the case of (10), if we plausibly prescribe probabilities, the second part of the first clause will not be satisfied (and neither will the second clause).

As we have seen, contrary to TPC, my theory accepts Dutchman conditionals but not irrelevant ones—(10)-like. If we take into consideration the way we use conditionals it seems to be an improvement.

4. Embedded Conditionals

Embedded conditionals are conditionals inside more complex sentences. They are also, arguably, the hardest cases for probabilistic theories of conditionals. In this section, I will discuss the embeddings of conditionals divided into two groups: conditionals embedded in probabilistic and non-probabilistic contexts.

¹⁴ If b) is satisfied we cannot compute the value of conditional probability by means of RF. That is why TPC does not give us any results in such cases.

¹⁵ It is easy to check that this assumption does not change the result of the test, as long as the other ratios are preserved.

By the probabilistic contexts, I will understand the contexts which, when supplemented by a sentence, gain the logical value on the basis of the probability of the embedded sentence. An example could be:

(13) The probability that x is 0.5.

The logical value of the sentence which we obtain by the substitution will depend on the probability of the sentence which we substitute, so if we put:

(14) The outcome of the next toss with that two-euro coin will be heads.

In the place of x the whole sentence will be true (assuming that the coin is unbiased). The different probabilistic contexts are conditionals in light of probabilistic theories. They gain logical value on the basis of the probabilities of their arguments. I will deal with such compound conditionals in Section 4.2.

By non-probabilistic contexts I mean, quite unsurprisingly, contexts that are not probabilistic in the above sense. It is impossible to discuss conditionals in all such contexts. In the next subsection, I will focus mainly on extensional contexts.

4.1. Conditionals in Non-probabilistic Contexts

Examples of conditionals in non-probabilistic contexts are:

(15) It is true that if you press this button the fire alarm goes off. [(548)c.*]

(16) If Martha is in the kitchen, we will have dinner soon, and if Marv is in the garage, the car will be fixed tonight (Kaufmann, 2009, p. 2).

Embeddings of this type are problematic for theories that deny that conditionals have truth values (e.g., TPC). If there is no truth value for a conditional inside, e.g., conjunction, how can we determine the truth value of the whole sentence? Still, there are possible strategies for explaining such occurrences. One of them was presented in Edgington's paper (1995). She claims that for all such embeddings it is possible to express their meanings without using embedded conditionals. Sadly, the scope of this strategy is limited. It was diagnosed in (Kölbel, 2000). It seems that a similar translation is not available if a conditional is embedded within the scope of existential quantification, for example:

(17) There is a boy in my class who, if I criticize him, will get angry (Kölbel, 2000, p. 105).

In contrast to TPC, there are no problems with such embeddings in the proposed theory. According to it, conditionals are truth-apt and if we know the logical value of a given conditional we can, via truth conditions, compute the truth

value of the whole sentence. For example (17) will be true iff for one of the students from the relevant class it is true that:

(18) If I criticize him, he will get angry.

As I have already noted it is impossible to discuss all non-probabilistic contexts. Still, it is easy to see that all embeddings in truth-functional contexts are easy: we just check the truth value of an embedded conditional(s), and on the basis of the truth conditions of the complex sentence determine its truth value. The cases of embeddings in extensional, but not truth-functional contexts, plausibly will also not be problematic. Strategies similar to the one used in the case of (18) are probably available there. It is hard to say anything certain about different non-probabilistic contexts like belief contexts. What is important, the main obstacle (the lack of the truth value) that makes the embeddings of conditionals difficult for TPC has been removed. Thus, as far as I know, embedded conditionals are no longer more problematic than any other embedded true-apt sentences.

4.2. Conditionals as Arguments in Probabilistic Contexts

Conditionals can appear in probabilistic contexts. The most discussed of such embeddings are compound conditionals, i.e., the conditionals with conditional antecedents or (and) consequents.

The examples of such conditionals are:

(19) If this vase will crack if it is dropped on wood, it will shatter if it is dropped on marble (Kaufmann, 2009, p. 2).

(20) If that apple is poisonous, then if you eat it you will die.

(21) If the red light is on, then if you ride another 100 kilometres your gas tank will be empty.

(22) If this house is a listed building, then if they built on a verandah, they acted illegally. [(739)a.]

If we consider the above examples it seems that we systematically use compound conditionals for example to describe dispositions. For instance, (20) describes what happens if you eat a poisonous apple. It seems that this kind of use is in line with the connection intuition. There is a connection between the antecedent which prescribes a disposition to some object and the consequent, conditional with triggering conditions in the antecedent and the effect of a disposition in the consequent. If so, probabilistic theories seem to be, in principle, able to capture cases of true compound conditionals. To do this we need to define the probability of conditionals. If we had such a definition we would be able to com-

pute the probability of conditionals with conditional arguments just as we do it with, say, conjunction, inside of conditionals.¹⁶

4.2.1. Stalnaker hypothesis and its trivialization.

The first attempt to define the probability of conditional sentences aimed to do that in terms of the probability of its arguments. Unfortunately, it seems to be impossible. The most notable of such attempts is sometimes called the Stalnaker hypothesis:

$$\text{SH } \Pr(A \rightarrow B) = \Pr(B/A)$$

It was disproved by Lewis (1976). Moreover, Lewis by a generalization of his result shows that no similar attempt could succeed, i.e., there is no proposition x such that $\Pr(x) = \Pr(A \rightarrow B)$.¹⁷

The idea to define the probability of conditionals in terms of the probability of its arguments seems to be at least unpromising, even if we bracket the triviality proofs. In the case of all classical functors, we have such definitions. For example, we can define the probability of disjunction in terms of probability of its arguments, for example:

$$\text{PD } \Pr(A \vee B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)^{18}$$

It is easy to see that this is the case if we see it in light of the natural interpretation of the sentences about probability:

NI The probability of x is the probability that x is true.

Disjunction is a truth-functional functor. The truth of conjunction depends only on the truth of its arguments, so it is natural that the probability of the whole conjunction depends on the probability of its arguments. The story is identical for all truth-functional functors. This kind of analysis, on the other hand, is not available for the case of non-truth functional functors, e.g., we can not define $\Pr(\forall_x P(x))$ in terms of $\Pr(P(x))$.

¹⁶ It is easy to see that both TPC and the new version of theory can easily incorporate the conditionals of the form $(A \wedge B) \rightarrow C$ or $(A \vee B) \rightarrow C$. By means of rules like PD (see below) we can define the probability of conjunction, then we continue to compute just as in cases of conditionals with simple arguments.

¹⁷ The argument with much weaker assumptions and the same conclusion was provided by Hájek (1994). In his later article (2012), he used structural similarities between ST and AT, as we have seen the only difference is that one thesis defines the probability of a conditional and second its acceptability, to propose a trivialization-like argument against AT and therefore TPC.

¹⁸ Where $\Pr(A \cap B) = \Pr(B/A) \Pr(A) = \Pr(A/B) \Pr(B)$.

It seems then that if we assume that natural language conditionals are not truth-functional, then it would be at least highly surprising if an analysis such as ST would succeed. The only truth-functional candidate worth examining is material implication.¹⁹ In face of its failure, we have to accept that the adequate truth-functional truth conditions for conditionals do not exist.

Another other definition of probability for conditionals is suggested by NI. If we combine this general rule with the truth conditions presented in Section 3.2 we will obtain the following proposal:

$$\text{PC} \quad \Pr(A \rightarrow B) = \Pr(((B/A) > 0.75) \wedge (\Pr(B/A) > \Pr(B))) \vee (\Pr(A) = 0)$$

In order to use PC to compute truth values of compound conditionals, a suitable framework of second-order probability (e.g., Baron, 1987) is required. Otherwise, it is unclear how to interpret the PC and compute the probability of embedded conditionals. In light of that extending theory to be able to handle embedded conditionals goes beyond the scope of this paper.

5. Consequence of the New Theory

In this final section, I will discuss some of the consequences of my theory.

5.1. Counterexamples

There is one obvious class of simple conditionals that are used by speakers and at the same time will be systematically considered false by my theory, the so-called “biscuit conditionals”, for example:

(23) If you are hungry, there is a pie in the fridge. [(628)a.]

(24) I will be in the garden if you need me. [(627)l.]

Clearly, utterances like (23) and (24) have the forms of conditionals, and at the same time, they will be judged as false by my theory (they neither involve connection which would make the first clause true nor have impossible antecedents). So it seems that they could be seen as a counterexample to my theory. On the other hand, it seems plausible that in these cases we use false conditionals to communicate some unconditional content. In the case of (23) it could be:

(23') There is a pie in the fridge. (I am telling you this in case you are hungry). [(628)b.]

¹⁹ It is easy to see that there are sixteen possible truth tables for a functor with two arguments. If we compare each of them with our intuitions about conditionals, the truth table of the material implication will be the most adequate.

If that is really what we communicate by means of (23), which seems plausible, then such sentences are neither true conditionals nor counterexamples to my theory.

5.2. Logical Properties of New Conditional

The detailed analysis of the logical properties of newly defined conditionals goes beyond the scope of the paper. At the same time, we can point toward some more interesting and promising properties.

First of all, the modus ponens will not be universally valid. To see that consider the following story:

You lie in hospital because you are suffering from a serious and until now an incurable disease. Someday a nurse comes into your room and asks you if you want to take part in experimental therapy. You agreed to participate. She smiles and comments on your decision: "If you undergo the treatment, you will be just fine" (25).

If it is the case that the therapy in the story cures nine out of every ten patients who have it, does the nurse say the truth when she asserts 25? If your intuitions are like mine, you will answer in the affirmative. Moreover, if you use the new theory the result would be the same. Now we can use 25 and true sentences:

T You had the treatment.

to infer by MP that, you will be fine. But still, it can be the case that you are in the unlucky ten percent of the patients and you do not recover. This shows that there are instances of MP which do not guarantee the preservation of truth.

This may initially seem to be an unintuitive consequence but in, at least two respects, it is not so implausible. Firstly, if a conditional probability ($\Pr(xc/xa)$) of a given conditional equals 1, the inference is deductively valid. Secondly, if we adopt the probabilistic notion of validity developed in Adams' (1975) according to which an inference is valid if the probability of the conclusion is not lower than the probability of the premises, then all instances of MP are valid. This split between probabilistic and categorical validity can potentially be used to reconcile the intuitiveness of MP with proposed semantics. Working out the details of this solution goes beyond the scope of the paper.

At the same time, the theory will correctly predict valid and invalid instances of antecedent strengthening. In the case of the valid instance, for example,

- (26) a) If Maureen plays the piano after 11, the neighbors complain.
 b) If Maureen plays the piano after 11, and she is in her pajamas, the neighbors complain.

conditional probability of the antecedent given the consequent do not fall below the threshold necessary for conditional to be true after we add an additional con-

junct to the antecedent. On the other hand in the case of the invalid instance of AS, for example:

- (27) a) If Maureen plays the piano after 11, the neighbors complain.
 b) If Maureen plays the piano after 11 and the neighbors are not home, the neighbors complain.

the addition lowers the conditional probability below the required level. A similar explanation is available for valid and invalid instances of transitivity.

Finally, the proposed theory does not validate Conjunctive Sufficiency principle, also called centering:

CS $A \wedge B \models A \rightarrow B$

CS is an inference that takes us from a conjunction to the conditionals from one of the conjuncts to another one. It will not be validated by proposed semantics, the second clause of the condition a) will not be satisfied in the case of many conjunctions. The fact that two sentences happened to be true at the same time does not entail that the truth of one of them promotes the truth of the second one. CS is valid in many popular semantics such as possible world semantics (e.g., Stalnaker, 1968) or three-valued semantics (e.g., Baratgin, Politzer, Over, Takahashi, 2018; Égré, Rossi, Sprenger, 2021). At the same time, some of the instances of CS seem to be counter-intuitive. For example:

- (28) The clear sky is blue and Beijing is the capital of China. \models If the clear sky is blue, Beijing is the capital of China.

The conjunction is true but the conditional seems to be false, there seems to be no relation between the blueness of the sky and Beijing being the capital of China. Therefore (28) seems to be a counter-example to CS. But is CS supported by the results of empirical experiments? The results are mixed but overall they seem to go against the principle. Results of the experiment presented in (Cruz, Over, Oaksford, Baratgin, 2016) support CS by showing that the way participants reacted to instances of CS is more similar to how they typically react to valid rather than invalid inferences. On the other hand, the results of Krzyżanowska, Collins, Hahn (2017), Douven, Elqayam, Singmann, van Wijnbergen-Huitink (2020), and Skovgaard-Olsen, Kellen, Hahn, Klauer (2019) go against the CS. For example, results of Krzyżanowska, Collins and Hahn (2017) suggest that the speakers expect a stronger connection between the arguments of a true conditional than between the conjuncts of true conjunction. This strongly suggests that centering is not a valid principle. The proposed semantics is not the only one that does not validate the centering, another such theory is, already mentioned, inferential semantics (e.g., Douven, Elqayam, Krzyżanowska, 2022). On the other hand, given that many of the prominent proposals validate the cen-

tering and it is not supported by the available empirical evidence it seems to be another attractive feature of the proposal.

Together with the fact that the theory incorporates the probabilistic relevance condition and explains the connection intuition, these features make the theory uniquely attractive. None of the most popular alternative theories such as possible world semantics or three-valued semantics can incorporate a similar relevance condition. Therefore, in opposition to the new theory, they are not able to explain why we do not like (10)-like conditionals.

6. Conclusion

In this concluding section, I will describe, a place of my theory in the literature devoted to indicative conditionals and gaps in my analysis which will be filled by future studies.

A similar theory was proposed by Douven (2008). The core claim of his theory, as presented in a later article (Douven, Verbrugge, 2012), is:

EST An indicative conditional “If A , B ” is assertable/acceptable if and only if $\Pr(B|A)$, is not only high but also higher than $\Pr(B)$.

Both theories use the main clause which requires a conditional to have a high conditional probability of a consequent given an antecedent and the second clause which requires that an antecedent is probabilistically relevant to the consequent. Here similarities end. The main difference is that Douven’s theory is pragmatic: it defines the acceptability of conditionals. Because of this difference, both theories are not competitors. If so, what is the relation between them? In the latter article, the authors do not commit themselves to any view concerning an explanation of their pragmatics: “Douven is noncommittal on whether EST is a brute fact about indicatives, or whether it follows from their truth conditions, ‘or from pragmatic principles like the Gricean maxims of good conversation’, or from something else altogether” (Douven, Verbrugge, 2012, p. 486).

After some minor adjustments,²⁰ my theory can be used to explain, in an elegant way, the proposed acceptability conditions along the line sketched in the second disjunct.

The theory is not complete. As we have seen the logical properties have to be worked out, the extension to nested conditionals is another task. I am confident that all these gaps will be filled in follow-up studies.

²⁰ One modification would have to involve an attitude towards Dutchman conditionals. I include them in my analysis and add to TC a sub-clause that addresses them. At the same time, Douven excludes them from his analysis.

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